Integrate by Partial Fractions

Use the method of partial fractions to compute the integral:

$$\frac{x^2 + 2x + 3}{(x+1)(x+2)(x+3)}\,dx.$$

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$$\frac{x^2+2x+3}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$$

$x = -1$:

$$\frac{(-1)^2+2(-1)+3}{(-1+2)(-1+3)} = A$$

$$A = \frac{1-2+3}{2}$$

$$= 1$$

$x = -2$:

$$B = \frac{4-4+3}{(-1)(1)}$$

$$= -3$$

$x = -3$:

$$C = \frac{9-6+3}{(-2)(-1)}$$

$$= 3$$

$$\therefore \int \frac{x^2+2x+3}{(x+1)(x+2)(x+3)}\,dx = \int \frac{1}{x+1}\,dx - \int \frac{3}{x+2}\,dx + \int \frac{3}{x+3}\,dx$$

$$= \ln|x+1| - 3\ln|x+2| + 3\ln|x+3| + C$$